

## 5.1.1 EXERCISES

For a link to all of the additional resources available for this section, click [OSttS Chapter 5 materials](#).

In Exercises 1 - 12, use the given pair of functions to find the following values if they exist.

- $(g \circ f)(0)$                                       •  $(f \circ g)(-1)$                                       •  $(f \circ f)(2)$
- $(g \circ f)(-3)$                                       •  $(f \circ g)\left(\frac{1}{2}\right)$                                       •  $(f \circ f)(-2)$

For help with these exercises, click the resource below:

- [Finding the composition of two functions](#)

- |   |   |
|---|---|
| 1. $f(x) = x^2$ , $g(x) = 2x + 1$                         | 2. $f(x) = 4 - x$ , $g(x) = 1 - x^2$                      |
| 3. $f(x) = 4 - 3x$ , $g(x) =  x $                         | 4. $f(x) =  x - 1 $ , $g(x) = x^2 - 5$                    |
| 5. $f(x) = 4x + 5$ , $g(x) = \sqrt{x}$                    | 6. $f(x) = \sqrt{3 - x}$ , $g(x) = x^2 + 1$               |
| 7. $f(x) = 6 - x - x^2$ , $g(x) = x\sqrt{x + 10}$         | 8. $f(x) = \sqrt[3]{x + 1}$ , $g(x) = 4x^2 - x$           |
| 9. $f(x) = \frac{3}{1 - x}$ , $g(x) = \frac{4x}{x^2 + 1}$ | 10. $f(x) = \frac{x}{x + 5}$ , $g(x) = \frac{2}{7 - x^2}$ |
| 11. $f(x) = \frac{2x}{5 - x^2}$ , $g(x) = \sqrt{4x + 1}$  | 12. $f(x) = \sqrt{2x + 5}$ , $g(x) = \frac{10x}{x^2 + 1}$ |

In Exercises 13 - 24, use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation.

- $(g \circ f)(x)$                                       •  $(f \circ g)(x)$                                       •  $(f \circ f)(x)$

For help with these exercises, click one of the the resources below:

- [Finding the composition of two functions](#)
- [Finding the domain of a composite function](#)

- |  |  |
|--|--|
| 13. $f(x) = 2x + 3$ , $g(x) = x^2 - 9$   | 14. $f(x) = x^2 - x + 1$ , $g(x) = 3x - 5$   |
| 15. $f(x) = x^2 - 4$ , $g(x) =  x $      | 16. $f(x) = 3x - 5$ , $g(x) = \sqrt{x}$      |
| 17. $f(x) =  x + 1 $ , $g(x) = \sqrt{x}$ | 18. $f(x) = 3 - x^2$ , $g(x) = \sqrt{x + 1}$ |

19.  $f(x) = |x|, g(x) = \sqrt{4-x}$

20.  $f(x) = x^2 - x - 1, g(x) = \sqrt{x-5}$

21.  $f(x) = 3x - 1, g(x) = \frac{1}{x+3}$

22.  $f(x) = \frac{3x}{x-1}, g(x) = \frac{x}{x-3}$

23.  $f(x) = \frac{x}{2x+1}, g(x) = \frac{2x+1}{x}$

24.  $f(x) = \frac{2x}{x^2-4}, g(x) = \sqrt{1-x}$

In Exercises 25 - 30, use  $f(x) = -2x$ ,  $g(x) = \sqrt{x}$  and  $h(x) = |x|$  to find and simplify expressions for the following functions and state the domain of each using interval notation.

25.  $(h \circ g \circ f)(x)$

26.  $(h \circ f \circ g)(x)$

27.  $(g \circ f \circ h)(x)$

28.  $(g \circ h \circ f)(x)$

29.  $(f \circ h \circ g)(x)$

30.  $(f \circ g \circ h)(x)$

In Exercises 31 - 40, write the given function as a composition of two or more non-identity functions. (There are several correct answers, so check your answer using function composition.)

31.  $p(x) = (2x+3)^3$

32.  $P(x) = (x^2 - x + 1)^5$

33.  $h(x) = \sqrt{2x-1}$

34.  $H(x) = |7-3x|$

35.  $r(x) = \frac{2}{5x+1}$

36.  $R(x) = \frac{7}{x^2-1}$

37.  $q(x) = \frac{|x|+1}{|x|-1}$

38.  $Q(x) = \frac{2x^3+1}{x^3-1}$

39.  $v(x) = \frac{2x+1}{3-4x}$

40.  $w(x) = \frac{x^2}{x^4+1}$

41. Write the function  $F(x) = \sqrt{\frac{x^3+6}{x^3-9}}$  as a composition of three or more non-identity functions.

42. Let  $g(x) = -x$ ,  $h(x) = x+2$ ,  $j(x) = 3x$  and  $k(x) = x-4$ . In what order must these functions be composed with  $f(x) = \sqrt{x}$  to create  $F(x) = 3\sqrt{-x+2} - 4$ ?

43. What linear functions could be used to transform  $f(x) = x^3$  into  $F(x) = -\frac{1}{2}(2x-7)^3 + 1$ ? What is the proper order of composition?

In Exercises 44 - 55, let  $f$  be the function defined by

$$f = \{(-3, 4), (-2, 2), (-1, 0), (0, 1), (1, 3), (2, 4), (3, -1)\}$$

and let  $g$  be the function defined

$$g = \{(-3, -2), (-2, 0), (-1, -4), (0, 0), (1, -3), (2, 1), (3, 2)\}$$

Find the value if it exists.

44.  $(f \circ g)(3)$

45.  $f(g(-1))$

46.  $(f \circ f)(0)$

47.  $(f \circ g)(-3)$

48.  $(g \circ f)(3)$

49.  $g(f(-3))$

50.  $(g \circ g)(-2)$

51.  $(g \circ f)(-2)$

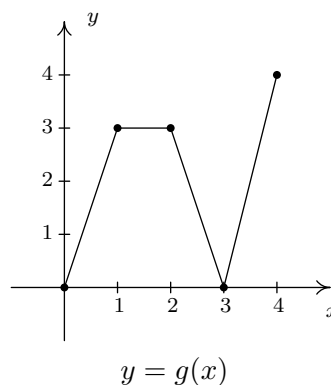
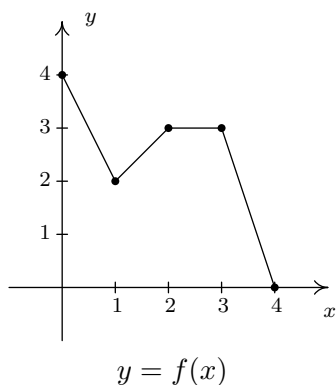
52.  $g(f(g(0)))$

53.  $f(f(f(-1)))$

54.  $f(f(f(f(f(1)))))$

55.  $\underbrace{(g \circ g \circ \cdots \circ g)}_{n \text{ times}}(0)$

In Exercises 56 - 61, use the graphs of  $y = f(x)$  and  $y = g(x)$  below to find the function value.



56.  $(g \circ f)(1)$

57.  $(f \circ g)(3)$

58.  $(g \circ f)(2)$

59.  $(f \circ g)(0)$

60.  $(f \circ f)(1)$

61.  $(g \circ g)(1)$

62. The volume  $V$  of a cube is a function of its side length  $x$ . Let's assume that  $x = t + 1$  is also a function of time  $t$ , where  $x$  is measured in inches and  $t$  is measured in minutes. Find a formula for  $V$  as a function of  $t$ .

63. Suppose a local vendor charges \$2 per hot dog and that the number of hot dogs sold per hour  $x$  is given by  $x(t) = -4t^2 + 20t + 92$ , where  $t$  is the number of hours since 10 AM,  $0 \leq t \leq 4$ .
- (a) Find an expression for the revenue per hour  $R$  as a function of  $x$ .
  - (b) Find and simplify  $(R \circ x)(t)$ . What does this represent?
  - (c) What is the revenue per hour at noon?
64. Discuss with your classmates how ‘real-world’ processes such as filling out federal income tax forms or computing your final course grade could be viewed as a use of function composition. Find a process for which composition with itself (iteration) makes sense.

### Checkpoint Quiz 5.1

1. Let  $f(x) = \sqrt{x-2}$  and  $g(x) = \frac{x^2}{x^2-1}$ .
- (a) Find the domain of  $f \circ g$  and find and simplify a formula for  $(f \circ g)(x)$ .
  - (b) Find the domain of  $g \circ f$  and find and simplify a formula for  $(g \circ f)(x)$ .
2. Write  $F(x) = \sqrt{7x+5}$  as a composition of two or more non-identity functions.

For worked out solutions to this quiz, click the links below:

- [Quiz Solution Part 1](#)
- [Quiz Solution Part 2](#)

## 5.1.2 ANSWERS

1. For  $f(x) = x^2$  and  $g(x) = 2x + 1$ ,

- $(g \circ f)(0) = 1$                       •  $(f \circ g)(-1) = 1$                       •  $(f \circ f)(2) = 16$
- $(g \circ f)(-3) = 19$                       •  $(f \circ g)\left(\frac{1}{2}\right) = 4$                       •  $(f \circ f)(-2) = 16$

2. For  $f(x) = 4 - x$  and  $g(x) = 1 - x^2$ ,

- $(g \circ f)(0) = -15$                       •  $(f \circ g)(-1) = 4$                       •  $(f \circ f)(2) = 2$
- $(g \circ f)(-3) = -48$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \frac{13}{4}$                       •  $(f \circ f)(-2) = -2$

3. For  $f(x) = 4 - 3x$  and  $g(x) = |x|$ ,

- $(g \circ f)(0) = 4$                       •  $(f \circ g)(-1) = 1$                       •  $(f \circ f)(2) = 10$
- $(g \circ f)(-3) = 13$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \frac{5}{2}$                       •  $(f \circ f)(-2) = -26$

4. For  $f(x) = |x - 1|$  and  $g(x) = x^2 - 5$ ,

- $(g \circ f)(0) = -4$                       •  $(f \circ g)(-1) = 5$                       •  $(f \circ f)(2) = 0$
- $(g \circ f)(-3) = 11$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \frac{23}{4}$                       •  $(f \circ f)(-2) = 2$

5. For  $f(x) = 4x + 5$  and  $g(x) = \sqrt{x}$ ,

- $(g \circ f)(0) = \sqrt{5}$                       •  $(f \circ g)(-1)$  is not real                      •  $(f \circ f)(2) = 57$
- $(g \circ f)(-3)$  is not real                      •  $(f \circ g)\left(\frac{1}{2}\right) = 5 + 2\sqrt{2}$                       •  $(f \circ f)(-2) = -7$

6. For  $f(x) = \sqrt{3 - x}$  and  $g(x) = x^2 + 1$ ,

- $(g \circ f)(0) = 4$                       •  $(f \circ g)(-1) = 1$                       •  $(f \circ f)(2) = \sqrt{2}$
- $(g \circ f)(-3) = 7$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \frac{\sqrt{7}}{2}$                       •  $(f \circ f)(-2) = \sqrt{3 - \sqrt{5}}$

7. For  $f(x) = 6 - x - x^2$  and  $g(x) = x\sqrt{x+10}$ ,

- $(g \circ f)(0) = 24$                       •  $(f \circ g)(-1) = 0$                       •  $(f \circ f)(2) = 6$
- $(g \circ f)(-3) = 0$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \frac{27-2\sqrt{42}}{8}$                       •  $(f \circ f)(-2) = -14$

8. For  $f(x) = \sqrt[3]{x+1}$  and  $g(x) = 4x^2 - x$ ,

- $(g \circ f)(0) = 3$                       •  $(f \circ g)(-1) = \sqrt[3]{6}$                       •  $(f \circ f)(2) = \sqrt[3]{\sqrt[3]{3}+1}$
- $(g \circ f)(-3) = 4\sqrt[3]{4} + \sqrt[3]{2}$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \frac{\sqrt[3]{12}}{2}$                       •  $(f \circ f)(-2) = 0$

9. For  $f(x) = \frac{3}{1-x}$  and  $g(x) = \frac{4x}{x^2+1}$ ,

- $(g \circ f)(0) = \frac{6}{5}$                       •  $(f \circ g)(-1) = 1$                       •  $(f \circ f)(2) = \frac{3}{4}$
- $(g \circ f)(-3) = \frac{48}{25}$                       •  $(f \circ g)\left(\frac{1}{2}\right) = -5$                       •  $(f \circ f)(-2)$  is undefined

10. For  $f(x) = \frac{x}{x+5}$  and  $g(x) = \frac{2}{7-x^2}$ ,

- $(g \circ f)(0) = \frac{2}{7}$                       •  $(f \circ g)(-1) = \frac{1}{16}$                       •  $(f \circ f)(2) = \frac{2}{37}$
- $(g \circ f)(-3) = \frac{8}{19}$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \frac{8}{143}$                       •  $(f \circ f)(-2) = -\frac{2}{13}$

11. For  $f(x) = \frac{2x}{5-x^2}$  and  $g(x) = \sqrt{4x+1}$ ,

- $(g \circ f)(0) = 1$                       •  $(f \circ g)(-1)$  is not real                      •  $(f \circ f)(2) = -\frac{8}{11}$
- $(g \circ f)(-3) = \sqrt{7}$                       •  $(f \circ g)\left(\frac{1}{2}\right) = \sqrt{3}$                       •  $(f \circ f)(-2) = \frac{8}{11}$

12. For  $f(x) = \sqrt{2x+5}$  and  $g(x) = \frac{10x}{x^2+1}$ ,

- $(g \circ f)(0) = \frac{5\sqrt{5}}{3}$                       •  $(f \circ g)(-1)$  is not real                      •  $(f \circ f)(2) = \sqrt{11}$
- $(g \circ f)(-3)$  is not real                      •  $(f \circ g)\left(\frac{1}{2}\right) = \sqrt{13}$                       •  $(f \circ f)(-2) = \sqrt{7}$

13. For  $f(x) = 2x + 3$  and  $g(x) = x^2 - 9$

- $(g \circ f)(x) = 4x^2 + 12x$ , domain:  $(-\infty, \infty)$
- $(f \circ g)(x) = 2x^2 - 15$ , domain:  $(-\infty, \infty)$
- $(f \circ f)(x) = 4x + 9$ , domain:  $(-\infty, \infty)$

14. For  $f(x) = x^2 - x + 1$  and  $g(x) = 3x - 5$

- $(g \circ f)(x) = 3x^2 - 3x - 2$ , domain:  $(-\infty, \infty)$
- $(f \circ g)(x) = 9x^2 - 33x + 31$ , domain:  $(-\infty, \infty)$
- $(f \circ f)(x) = x^4 - 2x^3 + 2x^2 - x + 1$ , domain:  $(-\infty, \infty)$

15. For  $f(x) = x^2 - 4$  and  $g(x) = |x|$

- $(g \circ f)(x) = |x^2 - 4|$ , domain:  $(-\infty, \infty)$
- $(f \circ g)(x) = |x|^2 - 4 = x^2 - 4$ , domain:  $(-\infty, \infty)$
- $(f \circ f)(x) = x^4 - 8x^2 + 12$ , domain:  $(-\infty, \infty)$

16. For  $f(x) = 3x - 5$  and  $g(x) = \sqrt{x}$

- $(g \circ f)(x) = \sqrt{3x - 5}$ , domain:  $[\frac{5}{3}, \infty)$
- $(f \circ g)(x) = 3\sqrt{x} - 5$ , domain:  $[0, \infty)$
- $(f \circ f)(x) = 9x - 20$ , domain:  $(-\infty, \infty)$

17. For  $f(x) = |x + 1|$  and  $g(x) = \sqrt{x}$

- $(g \circ f)(x) = \sqrt{|x + 1|}$ , domain:  $(-\infty, \infty)$
- $(f \circ g)(x) = |\sqrt{x} + 1| = \sqrt{x} + 1$ , domain:  $[0, \infty)$
- $(f \circ f)(x) = ||x + 1| + 1| = |x + 1| + 1$ , domain:  $(-\infty, \infty)$

18. For  $f(x) = 3 - x^2$  and  $g(x) = \sqrt{x + 1}$

- $(g \circ f)(x) = \sqrt{4 - x^2}$ , domain:  $[-2, 2]$
- $(f \circ g)(x) = 2 - x$ , domain:  $[-1, \infty)$
- $(f \circ f)(x) = -x^4 + 6x^2 - 6$ , domain:  $(-\infty, \infty)$

19. For  $f(x) = |x|$  and  $g(x) = \sqrt{4 - x}$

- $(g \circ f)(x) = \sqrt{4 - |x|}$ , domain:  $[-4, 4]$
- $(f \circ g)(x) = |\sqrt{4 - x}| = \sqrt{4 - x}$ , domain:  $(-\infty, 4]$
- $(f \circ f)(x) = ||x|| = |x|$ , domain:  $(-\infty, \infty)$

20. For  $f(x) = x^2 - x - 1$  and  $g(x) = \sqrt{x - 5}$

- $(g \circ f)(x) = \sqrt{x^2 - x - 6}$ , domain:  $(-\infty, -2] \cup [3, \infty)$
- $(f \circ g)(x) = x - 6 - \sqrt{x - 5}$ , domain:  $[5, \infty)$
- $(f \circ f)(x) = x^4 - 2x^3 - 2x^2 + 3x + 1$ , domain:  $(-\infty, \infty)$

21. For  $f(x) = 3x - 1$  and  $g(x) = \frac{1}{x+3}$

- $(g \circ f)(x) = \frac{1}{3x+2}$ , domain:  $(-\infty, -\frac{2}{3}) \cup (-\frac{2}{3}, \infty)$
- $(f \circ g)(x) = -\frac{x}{x+3}$ , domain:  $(-\infty, -3) \cup (-3, \infty)$
- $(f \circ f)(x) = 9x - 4$ , domain:  $(-\infty, \infty)$

22. For  $f(x) = \frac{3x}{x-1}$  and  $g(x) = \frac{x}{x-3}$

- $(g \circ f)(x) = x$ , domain:  $(-\infty, 1) \cup (1, \infty)$
- $(f \circ g)(x) = x$ , domain:  $(-\infty, 3) \cup (3, \infty)$
- $(f \circ f)(x) = \frac{9x}{2x+1}$ , domain:  $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 1) \cup (1, \infty)$

23. For  $f(x) = \frac{x}{2x+1}$  and  $g(x) = \frac{2x+1}{x}$

- $(g \circ f)(x) = \frac{4x+1}{x}$ , domain:  $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, \infty)$
- $(f \circ g)(x) = \frac{2x+1}{5x+2}$ , domain:  $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, 0) \cup (0, \infty)$
- $(f \circ f)(x) = \frac{x}{4x+1}$ , domain:  $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, -\frac{1}{4}) \cup (-\frac{1}{4}, \infty)$

24. For  $f(x) = \frac{2x}{x^2-4}$  and  $g(x) = \sqrt{1-x}$

- $(g \circ f)(x) = \sqrt{\frac{x^2-2x-4}{x^2-4}}$ , domain:  $(-\infty, -2) \cup [1 - \sqrt{5}, 2) \cup [1 + \sqrt{5}, \infty)$
- $(f \circ g)(x) = -\frac{2\sqrt{1-x}}{x+3}$ , domain:  $(-\infty, -3) \cup (-3, 1]$
- $(f \circ f)(x) = \frac{4x-x^3}{x^4-9x^2+16}$ , domain:  $(-\infty, -\frac{1+\sqrt{17}}{2}) \cup (-\frac{1+\sqrt{17}}{2}, -2) \cup (-2, \frac{1-\sqrt{17}}{2}) \cup (\frac{1-\sqrt{17}}{2}, \frac{-1+\sqrt{17}}{2}) \cup (\frac{-1+\sqrt{17}}{2}, 2) \cup (2, \frac{1+\sqrt{17}}{2}) \cup (\frac{1+\sqrt{17}}{2}, \infty)$

25.  $(h \circ g \circ f)(x) = |\sqrt{-2x}| = \sqrt{-2x}$ , domain:  $(-\infty, 0]$

26.  $(h \circ f \circ g)(x) = |-2\sqrt{x}| = 2\sqrt{x}$ , domain:  $[0, \infty)$

27.  $(g \circ f \circ h)(x) = \sqrt{-2|x|}$ , domain:  $\{0\}$

28.  $(g \circ h \circ f)(x) = \sqrt{|-2x|} = \sqrt{2|x|}$ , domain:  $(-\infty, \infty)$

29.  $(f \circ h \circ g)(x) = -2|\sqrt{x}| = -2\sqrt{x}$ , domain:  $[0, \infty)$

30.  $(f \circ g \circ h)(x) = -2\sqrt{|x|}$ , domain:  $(-\infty, \infty)$
31. Let  $f(x) = 2x + 3$  and  $g(x) = x^3$ , then  $p(x) = (g \circ f)(x)$ .
32. Let  $f(x) = x^2 - x + 1$  and  $g(x) = x^5$ ,  $P(x) = (g \circ f)(x)$ .
33. Let  $f(x) = 2x - 1$  and  $g(x) = \sqrt{x}$ , then  $h(x) = (g \circ f)(x)$ .
34. Let  $f(x) = 7 - 3x$  and  $g(x) = |x|$ , then  $H(x) = (g \circ f)(x)$ .
35. Let  $f(x) = 5x + 1$  and  $g(x) = \frac{2}{x}$ , then  $r(x) = (g \circ f)(x)$ .
36. Let  $f(x) = x^2 - 1$  and  $g(x) = \frac{7}{x}$ , then  $R(x) = (g \circ f)(x)$ .
37. Let  $f(x) = |x|$  and  $g(x) = \frac{x+1}{x-1}$ , then  $q(x) = (g \circ f)(x)$ .
38. Let  $f(x) = x^3$  and  $g(x) = \frac{2x+1}{x-1}$ , then  $Q(x) = (g \circ f)(x)$ .
39. Let  $f(x) = 2x$  and  $g(x) = \frac{x+1}{3-2x}$ , then  $v(x) = (g \circ f)(x)$ .
40. Let  $f(x) = x^2$  and  $g(x) = \frac{x}{x^2+1}$ , then  $w(x) = (g \circ f)(x)$ .
41.  $F(x) = \sqrt{\frac{x^3+6}{x^3-9}} = (h(g(f(x))))$  where  $f(x) = x^3$ ,  $g(x) = \frac{x+6}{x-9}$  and  $h(x) = \sqrt{x}$ .
42.  $F(x) = 3\sqrt{-x+2} - 4 = k(j(f(h(g(x)))))$
43. One possible solution is  $F(x) = -\frac{1}{2}(2x-7)^3 + 1 = k(j(f(h(g(x)))))$  where  $g(x) = 2x$ ,  $h(x) = x - 7$ ,  $j(x) = -\frac{1}{2}x$  and  $k(x) = x + 1$ . You could also have  $F(x) = H(f(G(x)))$  where  $G(x) = 2x - 7$  and  $H(x) = -\frac{1}{2}x + 1$ .
44.  $(f \circ g)(3) = f(g(3)) = f(2) = 4$
45.  $f(g(-1)) = f(-4)$  which is undefined
46.  $(f \circ f)(0) = f(f(0)) = f(1) = 3$
47.  $(f \circ g)(-3) = f(g(-3)) = f(-2) = 2$
48.  $(g \circ f)(3) = g(f(3)) = g(-1) = -4$
49.  $g(f(-3)) = g(4)$  which is undefined
50.  $(g \circ g)(-2) = g(g(-2)) = g(0) = 0$
51.  $(g \circ f)(-2) = g(f(-2)) = g(2) = 1$
52.  $g(f(g(0))) = g(f(0)) = g(1) = -3$
53.  $f(f(f(-1))) = f(f(0)) = f(1) = 3$
54.  $f(f(f(f(f(f(1)))))) = f(f(f(f(f(3)))) = f(f(f(f(-1)))) = f(f(0)) = f(1) = 3$
55.  $\underbrace{(g \circ g \circ \cdots \circ g)}_{n \text{ times}}(0) = 0$

56.  $(g \circ f)(1) = 3$

57.  $(f \circ g)(3) = 4$

58.  $(g \circ f)(2) = 0$

59.  $(f \circ g)(0) = 4$

60.  $(f \circ f)(1) = 3$

61.  $(g \circ g)(1) = 0$

62.  $V(x) = x^3$  so  $V(x(t)) = (t + 1)^3$

63. (a)  $R(x) = 2x$

(b)  $(R \circ x)(t) = -8t^2 + 40t + 184$ ,  $0 \leq t \leq 4$ . This gives the revenue per hour as a function of time.

(c) Noon corresponds to  $t = 2$ , so  $(R \circ x)(2) = 232$ . The hourly revenue at noon is \$232 per hour.